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RELAXATION MODEL FOR DESCRIBING THE STRAIN OF POROUS MATERIALS

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UDC 624.131+539.215

Plastic volume deformation characterizes the strain of porous media. Various models involving, in particular, the porosity concept, are used for its description. A survey of these models is given in [1]. Maxwell's nonlinear model [2] has been found useful for plastic shearing strain in the case of rapidly occurring processes. We propose that plastic volume deformation also be considered within the framework of the relaxation model. Relaxation equations of elastoplastic strain with plastic volume and shearing strain are derived. An example illustrating the determination of interpolation expressions for the equation of state and the volumetric relaxation time is given. The proposed model describes qualitatively the anomalous increase in the amplitude of the reflected wave, which has been detected experimentally (see, for instance, [3]).

Assume that the medium under consideration does not experience shearing strain and that the stress tensor in this medium is reduced to pressure. In this case, the strain values in the medium are determined only by changes in the density ρ, which, for the assigned field of velocities u_i, is found from the continuity equation

$$\partial\rho/\partial t + u_\alpha \partial\rho/\partial x_\alpha + \rho\partial u_\alpha/\partial x_\alpha = 0. \quad (1)$$

As is known [4], in the absence of shearing strain, the density is related to the principal values of the Hencky tensor of logarithmic strain h_i by the relationship $\rho = \rho_0 \exp(-h_1 - h_2 - h_3)$ (ρ_0 is the initial density of the medium). If there is only volume strain in the medium, then $h_1 = h_2 = h_3$ and $\ln(\rho/\rho_0)$, the compression logarithm, constitutes the measure of deformation.

Assume that the volume strain rate can be divided effectively into its elastic and plastic parts:

$$\frac{d}{dt}\sum_{i=1}^{3} h_i = \frac{\partial u_j}{\partial x_j} = \frac{d}{dt}\sum_{i=1}^{3} h_i^e + \frac{d}{dt}\sum_{i=1}^{3} h_i^p.$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 145-149, September-October, 1988. Original article submitted July 7, 1987.

Henceforth, $\frac{d}{dt}=\frac{\partial}{\partial t}+u_\alpha\frac{\partial}{\partial x_\alpha}$. The compression ρ/ρ_0 is decomposed into the product of the elastic $\rho/\rho_* = \exp(-h_1^e - h_2^e - h_3^e)$ and the plastic $\rho_*/\rho_0 = \exp(-h_1^p - h_2^p - h_3^p)$ compression: $(\rho/\rho_0) = (\rho/\rho_*)(\rho_*/\rho_0)$.

Thus, Eq. (1) is reduced to the expression

$$\frac{d\ln(\rho_*/\rho)}{dt}=\frac{\partial u_i}{\partial x_i}-\frac{d\ln(\rho_0/\rho_*)}{dt}=\frac{\partial u_i}{\partial x_i}-\psi. \tag{2}$$

We assume that the intrinsic energy of the medium depends on the effective elastic strain h_i^e (the superscript e will be henceforth omitted) and on the entropy. In our case, this dependence is given by $E(h_1, h_2, h_3, S) = E(\rho_*/\rho, S)$. Using the thermodynamic identity [4]

$$dE=E_{h_i}dh_i+E_S\,dS=-\frac{\rho}{\rho_*}\frac{\partial E}{\partial(\rho/\rho_*)}d(h_1+h_2+h_3)+E_S\,dS,$$

we obtain the following for the stress:

$$\sigma_i=\rho\frac{\partial E}{\partial h_i}=\rho\frac{\partial E}{\partial\ln(\rho_*/\rho)}=-p. \tag{3}$$

As usual, the temperature is calculated by means of the expression $T = \partial E/\partial S$.

For the intrinsic energy, the following equation holds:

$$\rho\partial E/\partial t+\rho u_\alpha\partial E/\partial x_\alpha+p\partial u_i/\partial x_i=0, \tag{4}$$

which follows from the law of conservation of energy.

We now denote $\xi = \ln(\rho_*/\rho)$, and Eq. (2) is then written thus:

$$d\xi/dt=\partial u_i/\partial x_i-\psi, \tag{5}$$

while expression (3) assumes the form

$$p=-\rho E_\xi. \tag{6}$$

We now obtain from Eq. (4)

$$0=\rho E_\xi\frac{d\xi}{dt}+\rho E_S\frac{dS}{dt}+p\frac{\partial u_i}{\partial x_i}=\rho E_\xi\frac{\partial u_\alpha}{\partial x_\alpha}-\rho E_\xi\psi+\rho E_S\frac{dS}{dt}-\rho E_\xi\frac{\partial u_\alpha}{\partial x_\alpha}.$$

Hence follows the entropy equation

$$E_S dS/dt=E_\xi\psi. \tag{7}$$

For the closure of the model, it is necessary to determine the rate of plastic volume strain ψ, for which we use the relaxation model. We assume that $\psi=\xi/\tau_V=[\ln(\rho_*/\rho)]/\tau_V=(h_1+h_2+h_3)/\tau_V$. Thus, Eq. (5) for the effective elastic volume strain assumes the form

$$d\xi/dt=\partial u_i/\partial x_i-\xi/\tau_V, \tag{8}$$

while the entropy equation (7) becomes

$$E_S dS/dt=\xi E_\xi/\tau_V. \tag{9}$$

In view of the second law of thermodynamics, it is necessary that the inequality ($E_S = T > 0$, $\tau_V > 0$) $\xi E_\xi \geq 0$ be satisfied.

One naturally assumes that the relaxation time τ_V is a function of the parameters of state for the medium, which, as will become clear later on, makes it possible to describe experimental data with a sufficiently high accuracy.

By adding the momentum equation to the above equations, we obtain a system for describing a medium characterized by inelastic volume variation:

$$\rho du_i/dt+\partial p/\partial x_i=0,\ d\xi/dt-\partial u_i/\partial x_i=-\xi/\tau_V,$$

$$\rho dE/dt + p\partial u_i/\partial x_i = 0,\ d\rho/dt + \rho\partial u_i/\partial x_i = 0\ (p = -\rho E_\xi). \tag{10}$$

A corollary of system (10) is the entropy equation (9). If inelastic volume as well as shearing strain occur in the medium, it is necessary to use simultaneously the relaxation model described above and the relaxation model for shearing strain [2].

We provide a system of equations with two plastic strain relaxation processes. For the sake of simplicity, we shall consider the variant where strain occurs along the three principal axes. The case where there is nonzero shearing strain can readily be generalized by using, for instance, [2]. Assuming that the strain tensor is diagonal like the stress tensor, we obtain the following system:

$$\rho\frac{du_i}{dt} - \frac{\partial\sigma_i}{\partial x_i} = 0,\quad \frac{dh_i}{dt} - \frac{\partial u_i}{\partial x_i} = -\frac{h_i - (h_1 + h_2 + h_3)/3}{\tau_\sigma} - \frac{h_1 + h_2 + h_3}{\tau_V},$$

$$\rho\frac{dE}{dt} - \sum_{i=1}^{3}\sigma_i\frac{\partial u_i}{\partial x_i} = 0,\quad \frac{d\rho}{dt} + \rho\frac{\partial u_i}{\partial x_i} = 0.$$

Here, $E(h_1, h_2, h_3, S)$ is the intrinsic energy density, $\sigma_i = \rho E_{h_i}$ is the stress tensor, h_i is the Hencky strain tensor (principal values), τ_V is the relaxation time of volume strain, and τ_σ is the relaxation time of shearing strain. A corollary of this system is the equation for entropy

$$E_S\frac{dS}{dt} = \frac{1}{\tau_\sigma}\sum_{i=1}^{3}E_{h_i}\left(h_i - \frac{h_1 + h_2 + h_3}{3}\right) + \frac{1}{3\tau_V}(E_{h_1} + E_{h_2} + E_{h_3}) \times$$

$$\times (h_1 + h_2 + h_3) = \frac{1}{\tau_\sigma}\sum_{i=1}^{3}\left(E_{h_i} - \frac{E_{h_1} + E_{h_2} + E_{h_3}}{3}\right)\left(h_i - \frac{h_1 + h_2 + h_3}{3}\right) + \frac{1}{3\tau_V}(E_{h_1} + E_{h_2} + E_{h_3})(h_1 + h_2 + h_3) = Q.$$

The intrinsic energy $E(h_1, h_2, h_3, S)$ must be chosen so that the inequality $Q \geq 0$ is satisfied. It should be emphasized once more that, for a satisfactory description of experimental data, it is necessary to consider that the relaxation time τ_V and τ_σ depend on the parameters of state of the medium.

Let us consider the example of determining the interpolation expression for τ_V. For this, we state the problem of hydrostatic compression of an element of the medium at a constant strain rate. We assume that the element of the medium experiences hydrostatic compression (dilation) at the constant strain rate $\dot{\varepsilon} = \partial u_1/\partial x_1 + \partial u_2/\partial x_2 + \partial u_3/\partial x_3$ and that the pressure is uniformly distributed over the element in question. Within the framework of these assumptions, system (10) is reduced to the following:

$$\frac{d\xi}{dt} = \dot{\varepsilon} - \frac{\xi}{\tau_V},\quad \frac{dS}{dt} = \frac{\xi E_\xi}{E_S\tau_V},\quad \frac{1}{\rho}\frac{d\rho}{dt} = -\dot{\varepsilon}. \tag{11}$$

We use the equation of state in the following form:

$$E = \frac{K_0}{\alpha(\alpha+1)}\left[\left(\frac{\rho}{\rho_*}\right)^\alpha - 1\right] + c_V T_0\left[\left(\frac{\rho}{\rho_*}\right)^\gamma e^{S/c_V} - 1\right] + \left(\frac{K_0}{\alpha+1} + \gamma c_V T_0\right)\left(\frac{\rho_*}{\rho} - 1\right). \tag{12}$$

The pressure and the temperature calculated on the basis of this equation of state are given by the expressions

$$p = \rho^2 E_\rho = \frac{\rho_* K_0}{\alpha+1}\left[\left(\frac{\rho}{\rho_*}\right)^{\alpha+1} - 1\right] + \rho_*\gamma c_V T_0\left[\left(\frac{\rho}{\rho_*}\right)^{\gamma+1} e^{S/c_V} - 1\right],\quad T = E_S = T_0\left(\frac{\rho}{\rho_*}\right)^\gamma e^{S/c_V}.$$

Here, K_0 (the square of the velocity of sound) and c_V (specific heat at constant volume) must be determined under conditions of elastic behavior of the medium, where relaxation processes do not play a part; the constants α and γ are the interpolation indices of nonlinearity of the medium; and T_0 is the initial temperature.

We provide an example illustrating the method of choosing the interpolation expression for the volume relaxation time. This expression is not universal and can be modified if necessary. We shall use a material characterized by the $p(\varepsilon)$ diagram, where $\varepsilon = \ln(\rho_0/\rho)$, which is typical for porous materials that can be compacted. This diagram is shown in Fig. 1 (curve 1). The volume relaxation time τ_V is assigned by the interpolation expression

$$\tau_V = \tau_V^0\left|\frac{p}{p_0}\right|^{-m}\exp\left(\theta\left(\left(\frac{\rho}{\rho_0}\right)^n - 1\right)\right). \tag{13}$$

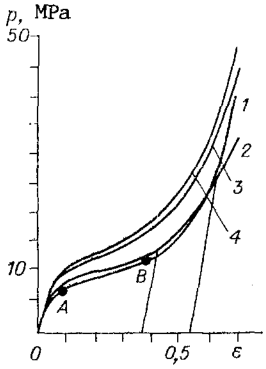

Fig. 1

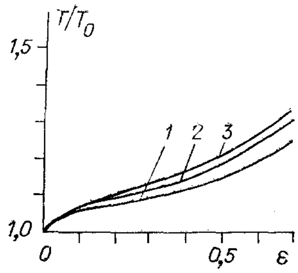

Fig. 2

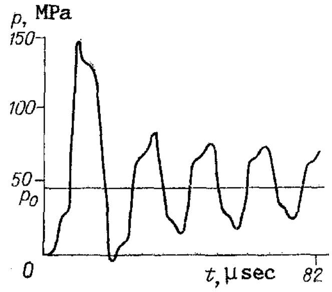

Fig. 3

The considerations on which this expression is based are the following: The multiplier $|p/p_0|^{-m}$ is responsible for the "shelf" of plastic strain AB in the $p(\varepsilon)$ diagram (the value of τ_V is small over this segment); the multiplier $\exp[\theta(\rho/\rho_0)^n - 1)]$ is responsible for the increase in the relaxation time for large compression values (the material becomes increasingly elastic, and the relaxation time increases).

System (11) was integrated numerically. The following values of the constants were used: $\rho_0 = 0.32$ g/cm^3, $K_0 = 0.47 \cdot 10^{10}$ cm^2/sec^2, $c_V = 0.35 \cdot 10^7$ cm^2/(sec^2·deg), $T_0 = 300$ K, $\alpha = 1$, $\gamma = 1$, $\tau_V^0 = 1$ sec, $p_0 = 7.52$ MPa, $m = 30$, $\theta = 13$, and $n = 2.2$.

Figure 1 shows the $p(\varepsilon)$ diagrams for the strain rates $\dot{\varepsilon} = -10^{-2}$; 10^2; 10^3 sec^{-1} (curves 2-4) and the $p(\varepsilon)$ diagrams corresponding to the load release for a material at the strain rate $\dot{\varepsilon} = 10^{-2}$ sec^{-1} from two points on the diagram of loading at the strain rate $\dot{\varepsilon} = 10^{-2}$ sec^{-1}. Figure 2 shows the $T(\varepsilon)$ diagram corresponding to loading at the same strain rates. It is evident that, for $\dot{\varepsilon} = -10^{-2}$ sec^{-1}, the $p(\varepsilon)$ curve describes satisfactorily the diagram defined by curve 1. A better fit can be obtained by varying the constants of the equation of state and the relaxation time.

We provide the results of numerical calculations revealing an anomalous increase in the amplitude of the wave reflected from a rigid wall. The essence of this phenomenon consists in the following: when a steady-state shock wave acts on a porous material layer lying on a rigid surface, the wave reflected from this surface has an amplitude much larger than that of the incoming wave (gas-dynamics theory predicts approximate doubling of the reflected wave amplitude). An experimental description of this phenomenon can be found, for instance, in [3].

The problem was solved for a layer with a thickness of 3 cm lying on a rigid base. We used the unidimensional variant of system (10) with the equation of state (12) and the relaxation time (13). The boundary conditions were assigned by the pressure $p = 45$ MPa for $x = 3$ cm and the zero velocity $u = 0$ for $x = 0$. S. K. Godunov's difference scheme [5] was used for solving the problem.

The calculation results have shown that, in this model, the compression wave is split into an elastic forerunner and a plastic wave. Figure 3 shows the pressure as a function of time at a rigid wall for $x = 0$. It is evident that the maximum amplitude of the reflected wave exceeds the pressure acting from the outside on the material layer by a factor larger than 3. The subsequent vibratory behavior of the pressure at a rigid wall qualitatively agrees with the experimental data given in [3].

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